Chapter 6

Conclusion

6.1 Further research

Copula-based modelling is an expanding area of research. The exploration of mixture copula models here provides a starting point for a number of further research opportunities.

Firstly, it is demonstrated here that any attempt to extend the parameter space of a copula through mixing will generally be unsuccessful due to the issue of identification. Further research is needed to elucidate the extent of this issue of identification in particular, and the issue of identification for parameter mixtures in general. Most discussions of parameter mixing focus on its ability to extend the parameter space, *à la* the *Beta-Binomial* distribution, but there is no general treatment of the circumstances in which parameters are not separately identifiable.

Secondly, it was remarked in relation to the NBA data that player position, or some other covariate, could determine the dependence structures concerned. Further research is needed on how copula-based models can be adapted to suit such situations, where the data set is composed of a number of subsets each with a potentially different dependence structure. One possibility is to model the dependence parameter by regression on a number of covariates. Parameter mixing, where the parameter is ascribed a distribution, can then form a starting point for such an exercise. Another possibility is to model the dependence structure using a finite mixture of copula models, each reflecting the dependence structure of one subset of the data. For the NBA data example, this means a finite mixture of copula models, each corresponding to one type of player positions in the data set. Further research is needed on whether such models can better describe these data.

6.2 Conclusion

A general extension to copula-based multivariate model construction is proposed and investigated in this thesis. Specifically, new copula-based models are generated through parameter mixing, and compared against their parent copulas in terms of modelling properties. This study makes a number of original contributions to the study of copula-based modelling.

Firstly, the properties of general parameter mixture models, other than uniform mixtures of shuffles of C (see page 14), are investigated. While the construction of new copulas through parameter-mixing has been studied in the past, their potential for modelling has not received a thorough investigation. Existing proposals for modelling using mixture copulas are scant, and where they exist, focus exclusively on one type — uniform mixtures of shuffles of C. By contrast, the focus in this thesis has been models constructed from parent copulas commonly used in statistical modelling, thus providing results relevant in modelling applications.

Secondly, the link between mixture copulas and their parent functions are explored. In past literature, once the mixture copula was constructed, the link to its parent copula has been ignored. We show, however, that this link can be exploited to easily extend certain properties of the parent copula to the mixture copula. For example, it is shown that an expression for the Spearman's rho measure of a mixture copula can be easily derived from that of the parent copula. Likewise, a simulation algorithm for the parent copula can be easily extended to that for the mixture copula. This is advantageous especially when a complex mixture model is to be constructed from a simple parent. While, for example, the simulation algorithm or Spearman's rho for the mixture copula are difficult to obtain from first principles, they can be easily found using the results for the simpler parent copula.

The other aspect of the link between parent and mixture copulas is a comparison between their modelling properties, in order to answer the question of whether and when parameter mixing yields modelling advantage. Firstly, it is shown here that attempts to extend the parameter space of copulas by introducing new parameters via mixing will generally be unsuccessful due to issues of identification. As a result, the parent and mixture copulas are mutually non-nested models. This is shown through an experiment, in which each of the mixture and parent copulas dominate the other when it is the true model. Likewise, the dependence coverage of the copula cannot be extended by mixing. In addition, the parent and mixture copulas become indistinguishable at the extremes of dependence coverage. This is confirmed via the experiment, where the greatest difference in modelling properties occur in the middle of the dependence coverage, with the differences diminishing towards the extremes.

The advantage of using a mixture model is demonstrated via the applied example using NBA data. It is demonstrated that the mixture model generally performs better than the parent. However, the degree of improvement varies. Improvement is most significant where the data exhibits small dependence.

Additionally, the use of mixing to convey prior information via "informative mixing" is examined and demonstrated. Because the mixture copula can be interpreted as a hierarchical model, prior information concerning the dependence parameter can be easily incorporated. This is demonstrated via the NBA example, where a partitioned model involving informatively mixed copulas outperform any model where the prior information is not used.

6.3 Note on appendices

An attached compact disc, titled *Modelling with parameter-mixture copulas - Interactive Supplement* forms the appendix to this document. Included are the relevant codes and data for this thesis, as well as an electronic copy of this thesis.